Midterm 1, Part 2

MAT 21D, Ragone

2021 Summer II

1. Each part of this exam is **45 minutes**. At the end of the 45 minutes, you will have 5 minutes to scan & upload your exam.
2. This exam is open note, open homework, and open book. You may **not** use any external electronic or online resource, nor may you work with others.
3. There are **6 questions**, each worth **15 points**. Where a question has multiple parts, the point breakdown has been listed.
4. You must show **all** your work. Answers without work will receive no credit.
5. You may either use physical pen & paper, or an electronic note-taking app. Answer each question on a new page. Please label continuing work by the number and "cont." (e.g., if problem 3 had 3 pages, you could write "3", "3, cont.", "3, cont. 2")
6. Clearly indicate which page goes with which problem. Upload all your pages to GradeScope, and correctly match each question with the pages.
7. Stay logged in to the Zoom room until you have uploaded your exam, in case you need technical assistance. **You do not need to keep your camera on.** If you are disconnected from the Zoom room for any reason, do not worry; message me through Canvas, or send me an e-mail, and continue to work on your midterm until I respond, or until time is up.

This exam is a chance to show off what you have learned so far. If you know how to do part of a question, write down what you know. Good luck!!

Problem 4

Given the below descriptions of some regions, write integrals that compute their areas/volumes. You do not need to evaluate these integrals. You are free to use whatever coordinate systems you prefer. (Note: it would be wise to sketch these regions, but I will not grade your sketches for this problem.)

(a) The region $R \subseteq \mathbb{R}^3$ bounded by the half-pipe $z = 1 - x^2$, the half-pipe $z = 1 - y^2$, and subject to $x, y, z \geq 0$.

(b) The "thick hemisphere". The volume above the xy plane between an inner sphere of radius R_{in} and an outer sphere of radius R_{out} with $R_{in} < R_{out}$.

Problem 5

(a) Write the change of coordinates between rectangular (x, y) and polar (r, θ) coordinates (more precisely, write the map $T : (r, \theta) \to (x, y)$), compute the Jacobian determinant of this map, and thus derive the relationship between the area elements $dxdy$ and $drd\theta$.

(b) The below integral of the function $f(x, y, z)$ over a cube is given in rectangular coordinates. Convert this integral to cylindrical coordinates. You do not need to evaluate this integral.

$$\int_0^1 \int_0^1 \int_0^1 f(x, y, z)\, dxdydz$$

Problem 6

Decide whether the below statements are true or false. In either case, you must reasonably argue why (note that counterexamples are logical reasons). Credit will only be awarded for proper reasoning.

(a) (T/F): Let us choose a density function $\sigma(x, y, z)$ for a spherical region $S \subseteq \mathbb{R}^3$ with radius R given by

$$\sigma(x, y, z) = \left|x^2 + y^2\right| + \left|z^2\right|$$

. Then using Fubini's theorem, we can calculate the mass by

$$\text{Mass} = \iiint_S \sigma \, dV = \int_0^{2\pi} \int_0^{\pi} \int_0^{R} \sigma(\rho, \phi, \theta)\rho^2 \sin\phi \, d\rho d\phi d\theta$$

(b) (T/F): There are no functions $f(x, y)$ which are discontinuous on the unit square $S \subseteq \mathbb{R}^2$ but still satisfy the equality

$$\int_0^1 \int_0^1 f(x, y) \, dxdy = \int_0^1 \int_0^1 f(x, y) \, dydx$$

(c) (T/F): The below integral computes the volume of a cylinder with radius $r = 1$ and height $h = 1$:

$$\int_0^{\pi} \int_0^1 \int_0^1 r \, drdzd\theta$$