Midterm 2, Part 2

MAT 21D, Ragone

2021 Summer II

1. Each part of this exam is **40 minutes**. At the end of the 40 minutes, you will have 10 minutes to scan & upload your exam.
2. This exam is open note, open homework, and open book. You may **not** use any external electronic or online resource, nor may you work with others.
3. There are **6 questions**, each worth **15 points**. Where a question has multiple parts, the point breakdown has been listed.
4. You must show **all** your work. Answers without work will receive no credit. Be specific when citing theorems.
5. You may either use physical pen & paper, or an electronic note-taking app. Answer each question on a new page. Please label continuing work by the number and "cont." (e.g., if problem 3 had 3 pages, you could write "3", "3, cont.", "3, cont. 2")
6. Clearly indicate which page goes with which problem. Upload all your pages to GradeScope, and correctly match each question with the pages.
7. Stay logged in to the Zoom room until you have uploaded your exam, in case you need technical assistance. **You do not need to keep your camera on.** If you are disconnected from the Zoom room for any reason, do not worry; message me through Canvas, or send me an e-mail, and continue to work on your midterm until I respond, or until time is up.

This exam is a chance to show off what you have learned so far. If you know how to do part of a question, write down what you know. Good luck!!

Problem 4

Compute a potential function for the vector field $\vec{F} = y\cos(x)\vec{i} + \sin(x)\vec{j}$.

Problem 5

Consider the following piecewise smooth oriented curve in $\mathbb{R}^3$: start at $A = (1, 0, 0)$, move along the quarter of the circle $x^2 + z^2 = 1$, $y = 0$, to reach $B = (0, 0, 1)$, and then travel back to A by a straight line.

(a) Sketch this curve.

(b) Write parameterizations $\vec{r}_1, \vec{r}_2$ for the sections of this curve.

(c) Compute the circulation along C for each of the following velocity fields $\vec{F}, \vec{G}$ (*Hint: you may want to think carefully about each vector field first*). Do not use Green's theorem.

(i) $\vec{F} = -z\vec{i} + 0\vec{j} + x\vec{k}$

(ii) $\vec{G} = yz\vec{i} + xz\vec{j} + xy\vec{k}$

Problem 6

Decide whether the below statements are true or false. In either case, you must reasonably argue why (note that counterexamples are logical reasons). Credit will only be awarded for proper reasoning. If you use a theorem, you must state what conditions are satisfied!

(a) (T/F): Consider the vector field $\vec{F} = \frac{-y}{x^2+y^2}\vec{i} + \frac{x}{x^2+y^2}\vec{j}$. Then $\vec{F}$ has a scalar potential $f(x, y) = \arctan(\frac{y}{x})$ with domain $D = \mathbb{R}^2$.

(b) (T/F): Suppose a vector field $\vec{F} = M\vec{i}+N\vec{j}+P\vec{k}$, where M, N, P have continuous first partial derivatives, has as its domain the unit sphere S^2, where a point (x, y, z) is in S^2 if $x^2 + y^2 + z^2 = 1$. If $\text{curl}(\vec{F}) = \vec{0}$ at all points on S^2, then $\vec{F}$ has a potential function f on S^2.

(c) (T/F): Let $\vec{F}(x, y, z) = M(x)\hat{i} + \alpha\hat{j} + \beta\hat{k}$ where $M(x)$ is a differentiable scalar function of one variable defined for all $x \in \mathbb{R}$ and $\alpha, \beta \in \mathbb{R}$ are constants. Then $\vec{F}$ is conservative.